- **5280:** *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain*

  Let $a \geq b \geq c$ be nonnegative real numbers. Prove that

$$\frac{1}{3}\left(\frac{(a+b)(c+a)}{2+\sqrt{a+b}}+\frac{(c+a)(b+c)}{2+\sqrt{c+a}}+\frac{(b+c)(a+b)}{2+\sqrt{b+c}}\right) \leq \frac{(a+b)^2}{2+\sqrt{b+c}}.$$

Solution 3 by Arkady Alt, San Jose, CA

Note that:

1. $c \leq b \iff c+a \leq a+b \iff \dfrac{(a+b)(c+a)}{2+\sqrt{a+b}} \leq \dfrac{(a+b)^2}{2+\sqrt{a+b}}$ and $c \leq a \iff 2+\sqrt{b+c} \leq 2+\sqrt{a+b} \iff \dfrac{(a+b)^2}{2+\sqrt{a+b}} \leq \dfrac{(a+b)^2}{2+\sqrt{b+c}}$ yields $\dfrac{(a+b)(c+a)}{2+\sqrt{a+b}} \leq \dfrac{(a+b)^2}{2+\sqrt{b+c}}$;

2. $\begin{cases} a+b \geq c+a \\ a+b \geq b+c \end{cases}$ $\dfrac{(c+a)(b+c)}{2+\sqrt{c+a}} \leq \dfrac{(a+b)^2}{2+\sqrt{c+a}}$ and $2+\sqrt{c+a} \geq 2+\sqrt{b+c}$ yields $\dfrac{(c+a)(b+c)}{2+\sqrt{c+a}} \leq \dfrac{(a+b)^2}{2+\sqrt{b+c}}$;

3. $\dfrac{(b+c)(a+b)}{2+\sqrt{b+c}} \leq \dfrac{(a+b)^2}{2+\sqrt{b+c}} \iff b+c \leq a+b \iff c \leq a.$

Then $\dfrac{1}{3}\left(\dfrac{(a+b)(c+a)}{2+\sqrt{a+b}}+\dfrac{(c+a)(b+c)}{2+\sqrt{c+a}}+\dfrac{(b+c)(a+b)}{2+\sqrt{b+c}}\right) \leq$
$\dfrac{1}{3} \cdot 3\ frac(a+b)^2 2+\sqrt{b+c} = \dfrac{(a+b)^2}{2+\sqrt{b+c}}.$